# Partial coherence in coupled photonic crystal vertical cavity laser arrays

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A classical, stochastic, coupled-oscillator model for the emergence of partial coherence in arrays of vertical cavity surface emitting lasers is presented. The spectra of the uncoupled lasers determine the second order coherence properties of the coupled system. Predictions of the resultant radiation and interference patterns are verified experimentally. © 2009 Optical Society of America

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Semiconductor laser arrays are of practical interest in communications, imaging, and other fields of optical engineering. Coherent operation of two-dimensional vertical cavity surface emitting laser (VCSEL) arrays has been demonstrated [1–4]. However, recent observations of photonic crystal (PhC) VCSEL arrays [5] that show partially coherent behavior [6] are not predicted theoretically in the literature. We report on optically coupled two-by-one array of photonic crystal VCSELs that are evanescently coupled and exhibit both in-phase [7] and out-of-phase [8] operations. Such arrays are amenable to treatment by coupled mode theory [9]. This approach, however, predicts the coherent operation of a dominant mode of the coupled system, not the partially coherent operation. A deterministic coupled-oscillator approach has proven fruitful in predicting and explaining the complicated temporal behavior of strongly coupled arrays [10]. The coupled-oscillator picture also yields a wealth of quasiperiodic and chaotic behaviors [11]. None of these approaches predicts the observed partially coherent steady state operation of the coupled VCSEL system [6].

In this Letter, a two-by-one array of lasers is analyzed in a classical, linear, stochastic, coupled-oscillator model. The resultant coherence matrix is calculated and partially coherent operation is predicted. The results of the stochastic coupled-oscillator model, namely, a shift in the spatial frequency of the fringes of the far-field interference pattern, are verified experimentally.

The VCSEL array is produced by fabricating two-dimensional PhC patterns into the top facet of a layered system of semiconductor materials with a thin planar active region optically confined between layers of alternating high and low index material (Bragg reflectors). Defects in the PhC define transversely coupled laser arrays [5,6,8]. The VCSELs are electrically pumped and may be addressed independently through segmented contacts to provide independent control of the power spectrum of each VCSEL as shown in the inset of Fig. 1. When the VCSELs are in close proximity and the power spectra overlap, the VCSELs couple and exhibit a far-field intensity pattern, such as might be observed in a Young's type experiment with polychromatic partially coherent illumination of the pinholes [12]. In this partially coherent mode of operation, the current to each of the VCSELs may be used to control the location of the peak intensity in the far-zone radiation. The independent current injection also provides a means to vary the degree of coherence between the lasers [6].

The intensity of the field on a far-zone observation screen in Young's experiment is determined by the cross correlation function of the field at the pinholes. The intenstity, up to factors associated with propagation, is given by the expression [12]

$$I \propto \Gamma_{11}(0) + \Gamma_{22}(0) + 2\Re\Gamma_{12}(\tau), \tag{1}$$

where $\tau$ is the time delay associated with the path length difference from each of the laser facets to the point of observation and $\Gamma_{ij}$ is the correlation function between the field at the $i$th and the $j$th pinholes. It will prove convenient to work not with $\Gamma_{ij}(\tau)$ but with the cross-spectral density, $W_{ij}(\omega)$, which is the Fourier transform of $\Gamma_{ij}$ with respect to $\tau$. It is known that $W_{ij}(\omega)$ may be taken to be a correlation function itself, the result of an ensemble average in the frequency domain, $W_{ij}(\omega)=\langle U_i^*(\omega)U_j(\omega)\rangle$, where $U_i(\omega)$ is a frequency domain realization of a stationary random process [12].

In each realization of the ensemble of random fields, the output of each VCSEL is modeled as an os-

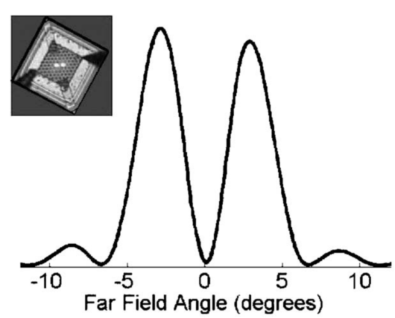


Fig. 1. Measured far-field intensity profile from two element arrays with 3.1 mA injected into each segment as in the inset showing the top view of the microcavity laser array.

 

cillator with amplitude $U_j(\omega)$ satisfying the frequency domain driven harmonic oscillator equation

$$f_j(\omega)U_j(\omega) = S_{j0}^{1/2} w_j(\omega), \tag{2}$$

where $S_{j0}^{1/2}$ is a constant, $f_j(\omega)$ characterizes the frequency response of the $j$th VCSEL, and $w_j(\omega)$ is a stochastic driving force. The VCSELs are assumed to be driven by uncorrelated white random processes. The frequency dependence of the cavity is thus responsible for producing the observed spectra. That is, $w_j(\omega)$ is a frequency domain realization of a stationary white complex random process with unity power spectrum, i.e., $\langle w_j^*(\omega) w_{j'}(\omega)\rangle = \delta_{jj'}$, where $S_{j0}$ is the maximum value of the power spectrum of the $j$th VCSEL and $f_j$ is a function normalized such that the maximum value of $|f_j(\omega)|^{-2}$ is unity. Thus, absent coupling between the devices, the cross-spectral density is given by the expression

$$W_{jj'}(\omega) = \langle U_j^*(\omega) U_{j'}(\omega)\rangle = S_{j0} |f_j(\omega)|^{-2} \delta_{jj'}. \tag{3}$$

To couple the VCSELs linearly, a driving term proportional to the field in the other cavity is added to each oscillator equation,

$$f_j(\omega)U_j(\omega) = S_{j0}^{1/2} w_j(\omega) + K U_{j'}(\omega), \quad j' \neq j, \tag{4}$$

where $K$ is a parameter proportional to the coupling. The elements of the $2\times 2$ cross-spectral density that emerge from this model are given by the expression

$$W(\omega) = |1 - K^2 f_1^{-1} f_2^{-1}|^{-2} \begin{pmatrix} |f_1|^{-2} S_{10} + |K f_1^{-1} f_2^{-1}|^2 S_{20} & K^* f_1^{-1*} |f_2|^{-2} S_{20} + K |f_1|^{-2} f_2^{-1} S_{10} \\ K f_1^{-1} |f_2|^{-2} S_{20} + K^* |f_1|^{-2} f_2^{-1*} S_{10} & |f_2|^{-2} S_{20} + |K f_2^{-1} f_1^{-1}|^2 S_{10} \end{pmatrix}, \tag{5}$$

where the argument $\omega$ is implied for $K$ and the $f_j$. The expression in Eq. (5) has a clear physical interpretation. The off-diagonal elements are proportional to $K$, and therefore the degree of spectral coherence goes to zero as $K$ goes to zero. The prefactor affects the spectral density but not the spectral degree of coherence. The prefactor may be understood to result from each field scattering multiple times between the cavities. To this end it is useful to recognize that

$$|1 - K^2 f_1^{-1} f_2^{-1}|^{-2} = \left| \sum_{n=0}^{\infty} (n+1)(K^2 f_1^{-1} f_2^{-1})^n \right|, \tag{6}$$

so that terms in the sum may be interpreted to represent multiple scattering between the cavities, each term of higher order being of narrower linewidth and thus more temporally coherent. Thus a stronger coupling will tend to produce stronger multiple scattering and therefore a greater degree of coherence in the time domain.

The coupled-oscillator model predicts a partially coherent operation of the array and greater temporally coherence with an increasing overlap of the spectra of the uncoupled lasers as well as with increasing coupling $K$. To relate the model cross-spectral density to the experiment, it is necessary to compute the time domain cross correlation matrix. For simplicity, assume that the uncoupled cavity fields produce Gaussian spectra so that

$$f_j^{-1} = e^{-(\omega - \omega_j)^2/4\Omega^2}. \tag{7}$$

Keeping only the first term in Eq. (6) it may be seen that the off-diagonal entries in the correlation matrix are of the form

$$\Gamma_{ij} = \frac{2\Omega}{\sqrt{3\pi}} e^{-\Delta\omega^2/6\Omega^2} e^{-4\tau^2\Omega^2/3} \times (K S_{j0} e^{-i\tau(2\omega_j+\omega_i)/3} + K^* S_{i0} e^{-i\tau(2\omega_i+\omega_j)/3}), \tag{8}$$

where $\Delta\omega = \omega_1 - \omega_2$. Further assuming that the spectra are of equal amplitude, $S_{i0} = S_{j0} = S_0$, it is seen that

$$\Re\Gamma_{12} \propto \cos(\bar{\omega}\tau + \phi_0)\cos(\Delta\omega\tau/6), \tag{9}$$

where $\bar{\omega} = (\omega_1 + \omega_2)/2$ and $\phi_0 = \arg K$.

Equation (9) provides a testable hypothesis for the model. It distinguishes the coupled-oscillator model from other linear mixing mechanisms. For example, if the VCSELs are assumed to be coupled through the driving terms so that $f_j(\omega)U_j(\omega) = S_{j0}^{1/2} w_j(\omega) + K S_{j'0}^{1/2} w_{j'}(\omega)$, then the elements of the $2\times 2$ cross-spectral density that emerge from this model are given by the expression

$$W(\omega) = \begin{bmatrix} |f_1|^{-2}(S_{10} + |K|^2 S_{20}) & f_1^{-1*} f_2^{-1}(K^* S_{20} + K S_{10}) \\ f_1^{-1} f_2^{-1*}(K S_{20} + K^* S_{10}) & |f_2|^{-2}(S_{20} + |K|^2 S_{10}) \end{bmatrix}. \tag{10}$$

This cross-spectral density exhibits the same characteristics as the coupled-oscillator model in that the spectral degree of coherence ranges between zero and unity depending on the value of $K$. However, assuming the spectra of the uncoupled lasers to be Gaussian as above, the off-diagonal elements of the cross correlation matrix are seen to be $\Re\Gamma_{12} \propto \cos(\bar{\omega}\tau + \phi_0)$. The dependence on $\Delta\omega$ is absent.

The off-diagonal elements of $\Gamma$ determine the fringe pattern seen in the far-field radiation pattern through Eq. (1). The dependence of $\Gamma$ on $\Delta\omega$ was tested by varying the pump current to one of the VCSELs. The VCSEL array, as described above, is constructed so that the quantum well under each VCSEL may be independently injected with current. The difference in frequency between modes of a VCSEL is expected to increase in a near-linear manner [13] with current. The current to one VCSEL was held at 3.1 mA while current to the second laser was varied. Above 6 mA, the frequency splitting was measured with an optical spectrum analyzer with a wavelength resolution of 0.06 nm. Closer to equal levels of current injection, the frequency splitting was inferred from the location of the far-field intensity minima. The relative time delay between points of observation in the far-zone is given by the expression $\tau = \sin(\theta) d/c$, where $d$ is the center-to-center distance between lasers and $\theta$ is the angular position (relative to the normal of the line connecting the VCSELS) in the far-zone. The minima in the beam pattern occur when $\bar{\omega}\tau + \phi = m\pi$; $m$ is an integer. For a standard Young's type experiment $\phi = \pm \pi/2$, or for the model in which only the source terms are mixed [resulting in Eq. (10)], $\phi = \arg K \pm \pi/2$. For the coupled-oscillator model, for $\bar{\omega} \gg \Delta\omega$, it may be seen from Eq. (9) that $\phi \approx \tau\Delta\omega/6 + \arg K \pm \pi/2$. The far-zone radiation pattern for one value of the current is shown in Fig. 1. Using the central minima surrounding the brightest fringes $\Delta\omega$ was calculated for the three points in Fig. 2 labeled "fringes." That is, $\tau$ and $\bar{\omega}$ are known and the extra phase accumulated was estimated from the position of the minima. It was observed that the central minimum appears between two peaks of nearly equal brightness, implying that the out-of-phase mode [14] is dominant ($\arg K \approx \pi/2$), although it is only necessary to assume that $\arg K$ is constant with respect to frequency (in the relatively narrow band of operation) to estimate the frequency difference $\Delta\omega$.

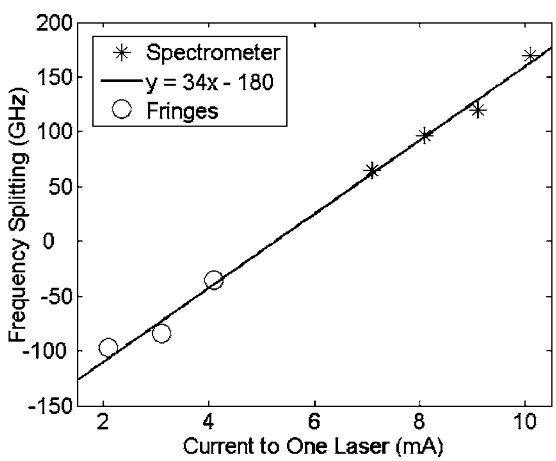


Fig. 2. Frequency difference between the VCSELS as a function of difference in current injection. The points for large current differences were measured by a spectrometer while at small current differences the frequency difference was computed from the diffraction pattern and Eq. (9).

It may be seen in Fig. 2 that the observations are in good agreement with the stochastic coupled-oscillator model together with a linear dependence of $\Delta\omega$ on injection current. That is, the VCSELs are coupled through the fields while subject to independent events of spontaneous emission. This suggests that the coherence properties of the coupled system might be controlled not just through the injection current but also by controlling the mode overlap or simply providing another channel to couple the field from one cavity to the other. For instance, it was recently shown that in a Young's experiment with pinholes in a metal screen, surface plasmon polaritons can dramatically alter the radiation pattern by cross coupling the fields at the pinholes [15]. A similar mechanism might be used in the design of coupled VCSEL arrays with tailored coherence properties by effectively controlling the coupling constant $K$. Moreover, this model predicts and explains the partially coherent operation of the array, unlike previous analyses. The theory presented here still requires the spectra of the uncoupled VCSELs and the coupling $K$ as the input. Future investigations will focus on a hybrid of the approach taken here together with a coupled mode calculation to obtain the spectra and coupling *ab initio* while retaining the partially coherent behavior.

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